

Time : 3 Hours

Max. Marks : 75

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. Evaluate $\int x^3 \sin x dx$.
2. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x dx$.
3. Evaluate $\int_0^1 \int_0^2 xy^2 dy dx$.
4. Find b_n in the expansion of $|\cos x|$ as a Fourier series in $(-\pi, \pi)$.
5. Solve $pq + p + q = 0$.
6. Solve $px^2 + qy^2 = z^2$.
7. Find $L[e^{3t} + e^{-3t}]$.
8. Prove that $L[\cosh at] = \frac{s}{s^2 - a^2}$.
9. If $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ then find $\text{div curl } \vec{F}$.
10. Show that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.

SECTION – B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. Obtain the reduction formula for $\int \sin^n x dx$.
12. Evaluate $\int_0^1 \int_0^2 \int_0^3 x + y + z dz dy dx$.
13. Find the Fourier series for $f(x) = x^2$ in $(-\pi, \pi)$.
14. Solve $p^2 + q^2 = x + y$.
15. Find the Laplace transform of $\sin 5t \sin 3t$.
16. Find $L^{-1} \left[\frac{s+3}{(s+1)(s-2)(s+2)} \right]$.
17. Show that $r^n \vec{r}$ is an irrotational vector for any value of n but is solenoidal only if $n = -3$.
18. Find the Directional derivatives of $\phi = x^2yz + 4xz^2$ at $(1,1,1)$ in the direction of $\vec{i} + \vec{j} + \vec{k}$.

SECTION – C (3 x 10 = 30)

Answer ALL the questions.

19. (a) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x dx$.

(Or)

(b) Find the Fourier series for $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$.

20. (a) Solve $(mz - ny)p + y(nx - lz)q = (ly - mx)$.

(Or)

(b) Solve by using Laplace transform $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$ given that $y(0) = -2$, $y'(0) = 5$.

21. (a) Find $(\nabla \cdot \vec{F})$, $(\nabla \times \vec{F})$ of the vector point function $\vec{F} = xz^3\vec{i} - 2x^2yz\vec{j} + 2yz^4\vec{k}$ at the point $(1, -1, 1)$.

(Or)

(b) Using Greens theorem show that $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy = 20$ where c is boundary of the rectangle formed by the line $x = 0$, $x = 1$, $y = 0$, $y = 2$.

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