Reg.No :

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL - 2019 **15CMA4A**

ABSTRACT ALGEBRA

Time : 3 Hours **Max. Marks : 75**

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. *In a group G, prove that* $\forall\, a \in G, (a^{-1})^{-1} = a$.
2. *If G has no nontrivial subgroups then show that G must be finite of prime order.*
3. *Give an example of a group in which right coset of a subgroup need not be equal to left coset.*
4. *Show that kernel of a homomorphism of a group G is a subgroup.*
5. *Find the orbits and cycles of* $\begin{pmatrix} 123456 \\ 654312 \end{pmatrix}$.
6. *Define an integral domain.*
7. *If* U *is an ideal of R and* $1 \in U$, *then prove that* $U = R$.
8. *Prove that every field is an integral domain.*
9. *Define a maximal ideal and give an example.*
10. *Prove that a Euclidean domain possesses a unit element.*

SECTION – B (5 x 5 = 25)

Answer any FIVE of the following questions.

11. *If G is a group in which* $(a \cdot b)^i = a^i \cdot b^i$ *for three consecutive integers* i *for all* a, b *in* G, *then show that G is abelian.*
12. *Show that every finite group of prime order is cyclic.*
13. *Prove that a subgroup N of a group G is a normal subgroup if and only if every left coset of N in G is a right coset of N in G.*
14. *Prove that every permutation is expressed product of 2 - cylces.*
15. *Prove that a finite integral domain is a field.*
16. *Let R be a Euclidean ring and* $a, b \in R$. *If* $b \neq 0$ *is not a unit in R, then prove that* $d(a) < d(ab)$.
17. *Let R be an Euclidean Ring then any element* $a, b \in R$ *have a greatest common divisor d. Moreover* $d = \lambda a + \mu b$ *for some* $\lambda, \mu \in R$.
18. *Let R be a commutative ring with unit element whose only ideals are (0) and R itself then prove R is a field.*

SECTION – C (3 x 10 = 30)

Answer ALL the questions.

19. (a) If G is a finite group and $a \in G$, then prove that $a^{o(G)} = e$, by proving the Lagrange's theorem.

(Or)

(b) Let G be a group of all 2x2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad - bc \neq 0$ and a, b, c, d are integers modulo 3, relative to matrix multiplication. Show that $o(G) = 48$.

20. (a) If H and K are finite subgroups of the abelian group of G of orders o(H) and o(K), respectively then show that $(HK) = \frac{o(H) \cdot o(K)}{o(H \cap K)}$.

(Or)

(b) State and prove Cayley's theorem.

21. (a) If U is an ideal of the ring R then prove that R/U is a ring and also show that it is a homomorphic image of R.

(Or)

(b) If R is a commutative ring with unit element and M is an ideal of R then M is a maximal ideal of $R \Leftrightarrow \frac{R}{M}$ is a field.

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