Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL – 2019 **15CPMA2A**

ALGEBRA – II

Time : 3 Hours **Max. Marks: 75**

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. (a) If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F moreover $[L : F] = [L : K][K : F]$.

(Or)

(b) If a, b in K are algebraic over F then prove that $a \pm b$, ab and $a/b\ (if\ b \neq 0)$ are all algebraic over F. In other words, the elements in K which are algebraic over F form a subfield of K.

2. (a) Prove that a polynomial of degree n over a field can atmost n roots in any extension field.

(Or)

(b) If $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, then prove that $F(v)$ is isomorphic to $F^{'}(w)$ where w is a root of $p^{'}(t)$ moreover, this isomorphism σ can so be chosen that

(i) $v\sigma = w$.

(ii) $\alpha\sigma = \alpha^{'}$ $for\ every\ \alpha \in F$.

3. (a) Prove that the fixed field of G is a subfield of K.

(Or)

(b) If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and its order, $O(G(K, F))$ satisfies $O(G(K, F)) \leq [K: F]$.

4. (a) Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by atmost n elements of G, for every integer n, then prove that G is a cyclic group.

(Or)

(b) If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F then prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

5. (a) Let C be the field of complex numbers and suppose that the division ring $D$is algebraic over C then prove that $D = C$.

(Or)

(b) If $p(x) \in F[x]$ is solvable by radicals over F, then prove that the Galois group over F of $p(x)$ is a solvable group.

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. *Prove that the number e is transcendental.*

7. *If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.*

8. *Let K be the splitting field of $f(x)$ in $F[x]$ and let $p(x)$ be an irreducible factor of $f(x)$ in $F[x]$. If the roots of $p(x)$ are $\alpha, \dots\dots, \alpha_r$ then prove that for each I there exists an automorphism σ_i in $G(K, F)$ such that $\sigma_i(\alpha_1) = \alpha_i$.*

9. *Let D be a division ring of characteristic $p > 0$ with center Z and let*

   $p = \{0, 1, 2, \dots\dots (p-1)\}$ be the subfield of Z isomorphic to J_p . Suppose that $a \in D$, $a \notin Z$ is such that $a^{p^n} = a$ for some $n \geq 1$ then prove that there exists an $x \in D$ such that

   (1) $xax^{-1} \neq a$.

   (2) $xax^{-1} \in p(a)$ the field obtained by adjoining a to p.

10. *Prove that every positive integer can be expressed as the sum of squares of four integers.*

* * * * * *