Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL – 2019 **15CPMA2B**

REAL ANALYSIS - II

Time : 3 Hrs **Max.Marks : 75**

SECTION-A (5x 6 =30)

Answer ALL the questions.

1. *(a) If f and g are in L(I) then the function* $|f|$ *in L(I) and prove that* $\left|\int_I f\right| \leq \int_I |f|$.

(Or)

(b) State and Prove Levi theorem for step function.

2. *(a) Let f be defined on I and assume that* $\{f_n\}$ *is a sequence of measurable function Such that* $f_n \to f$ *almost everywhere on I. Then f is measurable function on I.*

(Or)

(b) If $f \in L^2(I)$ *and* $g \in L^2(I)$ *then* $f.g \in L^2(I)$ *and* $\alpha f + \beta g \in L^2(I)$ *where* α, β *are real.*

3. *(a) State and prove Riemann lebesgue lemma.*

(Or)

(b) Let f be a real valued and continuous on a compact interval $[a, b]$*.Then for every* $\in > 0$ *there is a polynomial p such that* $|f(x) - p(x)| < \in$ *for every x in* $[a, b]$.

4. *(a) Define Total derivative and Prove that if f is differential at c then f is continuous at c.*

(Or)

(b) State and prove Taylors formula for function from R^n *to* R^1.

5. *(a) Let A be an open subset of* R^n *and assume that f:A→* R^n *has continuous partial derivatives* $D_j f_j$ *on A. If* $J_r(x) \neq 0$ *for all x in A then f is an open mapping.*

(Or)

(b) Assume that the second order partial derivatives $D_{i,j} f$ *exist in a ball B(a) and are continuous at a, where a is stationary point of f.*

Let $Q(t)=\frac{1}{2} f''(a, t) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} D_{i,j} f(a) t_i t_j$. *Then*

(i) *if* $Q(t) > 0$ $for\ all\ t \neq 0$ *then f has a relative minimum at a*

(ii) $Q(t) \lneqq 0$ $for\ all\ t \neq 0$ *then f has a relative maximum at a*

SECTION-B (3x15 =45)

Answer any THREE of the following questions.

6. *State and prove that Lebesgue dominated convergence theorem.*

7. *(i) State and prove that RIESZ –Fischer Theorem.*

   (ii) If S and T are measurable then so is S-T.

8. *(i) Define Fourier series*

   (ii) If g is bounded variation on [0,∞] then $lim_{\alpha\to\infty}\frac{2}{\pi}\int_0^\infty g(t)\frac{sin\alpha t}{t}\,dt = g(0+)$

9. *(i) State and prove mean value theorem*

   (ii) If g is differential at a and f is differential at $b = g(a)$ *then h=fog is differential at a.*

10. *State and prove that inverse function theorem.*

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