Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL– 2019 15CPMA2C PARTIAL DIFFERENTIAL EQUATIONS

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Time: 3 Hrs. Max.Marks : 75

SECTION–A(5x6=30)

Answer ALL the questions.

1. *(a)Find the integral surface of the linear PDE* $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ *containing the straight line* $x+y=0,\ z=1$.

(Or)

(b)Find the characteristics of the PDE $p^2+q^2=2$ *and determine the integral surface which passes through* $x=0,\ z=y$.

2. *(a)Derive Poisson equation.*

(Or)

(b) Find the steady state temperature distribution in a semicircular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U_0 *and its bounding diameter kept at zero temperature.*

3. *(a)In a one dimensional infinite solid* $-\infty<x<\infty$, *the surface* $a<x<b$ *is initially maintained at temperature* T_0 *and at zero temperature everywhere outside the surface show that*

$$T(x,t)=\frac{T_0}{2}\left[erf\left(\frac{b-x}{\sqrt{4\alpha t}}\right)-erf\left(\frac{a-x}{\sqrt{4\alpha t}}\right)\right]$$ *where erf is an error function.*

(Or)

(b)Find the temperature in sphere of radius a, when its surface is kept at zero temperature and its initial temperature is $f(r,\theta)$.

4. *(a)A stretched string of finite length L is held fixed at its ends and is subjected to an initial displacement* $u(x,0)=u_0\sin(\pi x/L)$. *The string is released from this position with zero initial velocity. Find the resultant time displacement motion of the string.*

(Or)

(b)Use Duhamel's principle to solve the heat equation problem described by

$$u_t(x,t)=ku_{xx}(x,t)+f(x,t), \qquad -\infty<x<\infty, t>0$$
$$u(x,0)=0 \qquad -\infty<x<\infty$$

5. *(a)Using Green's function technique to solve the Dirichlet's problem for a semi-infinite space.*

(Or)

(b)Solve the following IBVP using the Laplace transform technique:

$$PDE: u_t = u_{xx}, 0 < x < 1, t > 0$$

$$BCs: u(0,t) = 1, u(1,t) = 1, t > 0$$

$$IC: u(x,0) = 1 + \sin \pi x, 0 < x < 1$$

SECTION–B(3x15=45)

Answer any THREE of the following questions.

6. *Find the complete integral of* $(p^2+q^2)y = qz$.

7. *Find a general spherically symmetric solution of the following Helmholtz equation* $(\nabla^2 - k^2)u = 0$.

8. *A uniform rod of length L whose surface is thermally insulated is initially at temperature* $\theta = \theta_0$.

   At time $t = 0$, *one end is suddenly cooled to* $\theta = 0$ *and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution* $\theta(x,t)$.

9. *Derive the general periodic solution of one-dimensional wave equation in cylindrical coordinates.*

10. *Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function method and hence derive the Poisson integral formula.*

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