Reg No: | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL– 2019 15CPMA4A COMPLEX ANALYSIS – II

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Time: 3 Hrs. Max.Marks : 75

SECTION–A(5x6=30)

Answer ALL the questions.

1. *(a)Show that the Riemann Zeta function* $\zeta(s) = 2^s \pi^{s-1} sin\frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$.

   (Or)

   (b)Show that $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}(1 - p_n^{-s})$ *if* $Re(s) > 1$.

2. *(a)Establish Schwarz Christoffel formula.*

   (Or)

   (b) Establish Harnack's inequality.

3. *(a)Show that the zeros* $a_1, \ldots, a_n$ *and the poles* $b_1, \ldots, b_n$ *of an elliptic function satisfy* $a_1 + \cdots + a_n \equiv b_1 + \cdots + b_n (mod\ M)$.

   (Or)

   (b)If τ *is a complex number with* $Im(\tau) > 0$ *and* $\tau' = \frac{a\tau + b}{c\tau + d}$ *where a,b,c and d are integers with* $ad - bc = 1$, *prove that* $Im(\tau') > 0$.

4. *(a) Prove that* $\wp(z) - \wp(u) = -\frac{\sigma(z-u)\sigma(z+u)}{\sigma(z)^2\sigma(u)^2}$ *with usual notation.*

   (Or)

   (b) Derive Legendre's relation.

5. *(a)Show that two analytic continuation* $\bar{\gamma}_1$ *and* $\bar{\gamma}_2$ *of global analytic function* f *along the same arc* γ *are either identical or* $\bar{\gamma}_1(t) \neq \bar{\gamma}_2(t)$ *for all* t.

   (Or)

   (b) (i) Define a sheaf.

   (ii) Define Homotophic curves.

SECTION–B(3x15=45)

Answer any THREE of the following questions.

6. *State and prove Arzela's theorem.*
7. *State and prove Riemann mapping theorem.*
8. *Define basis of period module of* $f(z)$ *and show that any two bases of the same module are connected by a unimodular transformation.*
9. *Derive the differential equation satisfied by the weierstrass* $\wp$ *-function* $\wp(z)$.
10. *State and prove Monodromy theorem.*

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