

**D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1**  
**SEMESTER EXAMINATIONS**  
**APRIL – 2019**  
**MATHEMATICAL STATISTICS**

**15CPMA4B**

Time : 3 Hrs

Max. Marks : 75

**SECTION-A (5x6=30)**

Answer ALL the questions.

1. (a) Derive the probability density function of  $F$  – distribution.

(Or)

(b) Obtain the first four moments of  $t$ -distribution.

2. (a) Four coins were tossed at a time and this operation is repeated 160 times. It is found that 4 heads occur 6 times, 3 heads occur 43 times, 2 heads occur 69 times and one head occur 34 times.

Discuss whether the coins regarded as unbiased?

(Or)

(b) In a  $2 \times 2$  contingency table, prove that  $\chi^2 = \frac{(ad-bc)^2 N}{(a+b)(a+c)(b+d)(c+d)}$  where  $N = a + b + c + d$ .

3. (a) If  $X_1, X_2, \dots, X_n$  be random sample from the normal population with density function

$$f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} \sum (x_i - \mu)^2\right], -\infty < x < \infty, \text{ Find the maximum likelihood estimator of}$$

i)  $\mu$  When  $\sigma^2$  is known.    ii)  $\sigma^2$  When  $\mu$  is known

(Or)

(b) Show that  $S^2$  is not an unbiased estimate of the parameter  $\sigma^2$  for the class of  $D$  of distribution functions of random variables  $X$ .

4. (a) If  $X \sim N(\mu, 4)$ ,  $\mu$  unknown. To test  $H_0: \mu = -1$  against  $H_1: \mu = 1$  based on a sample of size 10 from this population and use the critical region  $x_1 + 2x_2 + 3x_3 + \dots + 10x_{10} \geq 0$ . Find the size of the critical region and power of the test.

(Or)

(b) Explain Analysis of variance for one-way classification.

5. (a) State and prove Fundamental identity.

(Or)

(b) Obtain the O.C function in testing a hypothesis concerning the expected value  $m$  of a normal population.

**SECTION-B (3x15 =45)**

Answer any THREE of the following questions.

6. Derive the probability density function of  $\chi^2$ -distribution. Also find Moment generating function hence find mean and variance.

7. Test the hypothesis of no difference between the ages of male and female employees of a certain company using Mann Whitney test for the sample data. Use 0.10 level of significance.

Males	3.1	2.5	3.8	3.3	4.2	4.0	4.4	2.6	4.3	3.5
Females	44	30	34	47	35	32	35	47	48	34

8. State and prove Rao – Crammer inequality.

9. The following table shows the yield per hectare of four different plant crops grown on lots treated with three different types of fertilizer. Test at 0.01 level of significance whether

- a) There is a significant difference in yield per hectare due to fertilizer
- b) There is a significant difference in yield per hectare due to crops.

Fertilizer \ Crop	Crop			
	I	II	III	IV
A	4.5	6.4	7.2	6.7
B	8.8	7.8	9.6	7.0
C	5.9	6.8	5.7	5.2

10. Give the Sequential probability ratio test for testing  $H_0: \theta = \theta_0$  against  $H_1: \theta = \theta_1 (> \theta_0)$  in sampling from a normal density  $f(x, \theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x-\theta}{\sigma} \right)^2 \right]$ ,  $-\infty < x < \infty$ . Where  $\sigma$  is unknown. Also obtain its O.C. function.

\* \* \* \* \*