Reg.No :

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

APRIL – 2019 **15CPMA4C**

FUNCTIONAL ANALYSIS

Time : 3 Hours **Max. Marks: 75**

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. *(a) State and prove Holder's inequality.*

(Or)

(b) Define a normed linear space and show that addition and scalar multiplication are jointly continuous.

2. *(a) State and prove Uniform boundedness theorem.*

(Or)

(b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

3. *(a) Let y be a fixed vector in a Hilbert space H and let f_y be a function defined as* $f_y(x) = (x, y) \, for \ every \ x \in H$. *Then prove that f_y is a functional on H and* $\|y\| = \|f_y\|$.

(Or)

(b) If T is an operator on H, then prove that T is normal if and only if its real and imaginary parts commute.

4. *(a) Define regular and singular elements of a Banach algebra. Prove that the set of all regular elements G of a Banach algebra is an open set.*

(Or)

(b) If x is an element in a Banach algebra, then prove that $r(x) = lim_{n\to\infty}\|x^n\|^{1/n}$.

5. *(a) Prove that there is a one to one correspondence of the set of all maximal ideals in a Banach algebra onto its set of all multiplicative functionals of A.*

(Or)

(b) If A is a Banach algebra without identity element, show that A can be extended to a Banach algebra B with an identity element.

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. a) *State and prove Hahn - Banach Theorem.*

   b) *Let* N *and* N' *be normed linear spaces and T is a linear transformation of* N *into* N'. *Then prove that the following are equivalent:*

   i) *T is continuous ;*

   ii) *T is continuous at the origin;*

   iii) *There exists a non - negative real number K with the property that* $\|T(x)\| \leq K\|x\|$ *for every* $x \in N$;

   iv) *If* $S = \{x : \|x\| \leq 1\}$ *is the closed unit sphere in N, then its image* $T(S)$ *is a bounded set in* N'.

7. a) *State and prove open mapping theorem.*

   b) *If M is a closed linear subspace of a Hilbert space H, then prove that* $H = M \oplus M^{\perp}$.

8. a) *State and prove Riesz representation theorem.*

   b) *If A is a positive operator on H then prove that* $I + A$ *is non - singular. Hence or otherwise show that* $I + T^{*}T$ *and* $I + TT^{*}$ *are non - singular.*

9. a) *Show that the boundary of the set of all singular elements(S) of a Banach algebra is subset of all topological divisors of zero(Z).*

   b) *Define a Banach algebra. If G is a finite group then show that its group algebra is the set of all complex functions defined on G.*

10. a) *State and prove Gelfand Neumark representation theorem.*

    b) *State the properties of Gelfand mapping.*

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