

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1
SEMESTER EXAMINATIONS
APRIL – 2019

15CPMA4D

DIFFERENCE EQUATIONS

Time : 3 Hours

Max. Marks: 75

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. (a) Show that the functions 3^n , $n3^n$, n^23^n are linearly independent on $n \geq 1$.

(Or)

(b) Solve the difference equation $(E - 3)(E + 2)y(n) = 5(3^n)$.

2. (a) Find A^n if $A = \begin{pmatrix} 0 & 1 & 1 \\ -2 & 3 & 1 \\ -3 & 1 & 4 \end{pmatrix}$.

(Or)

(b) A $k \times k$ matrix is diagonalizable iff it has k linearly independent Eigen vectors.

3. (a) Find the Z - transforms of the sequence $\{na^n\}$ and $\{n^2a^n\}$.

(Or)

(b) Solve: $x(n + 4) + 9x(n + 3) + 30x(n + 2) + 44x(n + 1) + 24x(n) = 0$,

$$x(0) = 0, x(1) = 0, x(2) = 1, x(3) = -10.$$

4. (a) Show that $t^2 \log t + t^3 = o(t^3)$ as $n \rightarrow \infty$.

(Or)

(b) Discuss variation of constant formula.

5. (a) Find the limit superior and the limit inferior for the following sequence.

(1) $s_1: 0, 1, 0, 1, \dots$

(2) $s_2: 1, -2, 3, -4, \dots$

(3) $s_3: 3/2, -1/2, 4/3, -1/3, 5/4, \dots$

(Or)

(b) If there exists a Sub sequence $b(n_k) \leq 0$ with $n_k \rightarrow \infty$ as $k \rightarrow \infty$ then every solution of

$$P(n)x(n + 1) + P(n - 1)x(n - 1) = b(n), x(n) \text{ Oscillates.}$$

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. *State and prove Limiting behaviour of solution.*

7. *Find the general solution of $x(n + 1) = Ax(n)$ with*

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix}.$$

8. *Find the zeros of $g(z) = z - A - B(z)$ all lie in the region $|z| < c$, for some real positive constant c , moreover $g(z)$ has finitely many zeros z with $|z| \geq 1$.*

9. *State and prove Poincare's theorem.*

10. *If $c(n) \geq a(n) \geq 0$ for all $n > 0$ and $z(n) > 0$ is a solution of*

$$c(n)z(n) + \frac{1}{z(n-1)} = 1 \text{ then the equation}$$

$$a(n)y(n) + \frac{1}{y(n-1)} = 1 \text{ has a solution } y(n) \geq z(n) \geq 1 \text{ for all } n \in \mathbb{Z}^+.$$

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