

CALCULUS OF VARIATIONS

| Semester | Subject Code | Category | Lecture | | Theory | | Practical | Credits |
|----------|--------------|----------------|----------|---------|----------|---------|-----------|---------|
| III | 21CPM A3B | Core – Paper X | Hrs/Week | Hrs/Sem | Hrs/Week | Hrs/Sem | 0 | 5 |
| | | | 6 | 90 | 6 | 90 | | |

COURSE OBJECTIVES:

The students will be able to

- Understand the foundations of calculus of variations and its Applications in mathematics and physics.
- Formulate variational problems and analyse them to deduce key properties of system behavior.

COURSE OUTCOMES:

On the successful completion of the course, the students will be able to

| CO Number | CO Statement | Knowledge Level (K1-K4) |
|-----------|--|-------------------------|
| CO1 | Use Euler-Lagrange equation or its first integral to find differential equations for stationary paths and solve simple initial and boundary value problems by using several independent variables of calculus. | K2 |
| CO2 | Understand the concept of differential, difference equation and Stochastic Calculus of Variations. | K2 |
| CO3 | Solve variational problems with a movable boundary for a functional dependent on two functions and reflection and refraction of extremals and diffraction of light rays | K3 |
| CO4 | Discuss the sufficient conditions for an extremum. | K4 |
| CO5 | Analyze the complementary variational principles, Poisson bracket, contact transformations, the Hamilton - Jacobi equation, Clairaut's theorem and Noether's theorem. | K4 |

Knowledge Level: K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze.

MAPPING WITH PROGRAMME OUTCOMES:

| COS | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 |
|-----|-----|-----|-----|-----|-----|-----|
| CO1 | S | M | M | M | S | M |
| CO2 | S | M | S | S | M | S |
| CO3 | M | M | S | S | M | S |
| CO4 | S | S | S | M | M | S |
| CO5 | M | S | M | M | S | M |

S – Strong; M – Medium; L - Low

UNIT - I: VARIATIONAL PROBLEMS WITH FIXED BOUNDARIES 18 Hours

The Concept of Variation and Its Properties- Euler's Equation- Variational Problems for Functionals of the Form- Functionals Dependent On Higher-Order Derivatives- Functionals Dependent on Functions of Several Independent Variables- Variational Problems in Parametric Form.

Chapter 1: Sections 1.1 - 1.6

UNIT - II: VARIATIONAL PROBLEMS WITH FIXED BOUNDARIES (CONTINUED)

18 Hours

Some Applications to Problems of Mechanics- Variational Problems Leading to an Integral Equation or a Differential-Difference Equation- Theorem of du Bois-Reymond- Stochastic Calculus of Variations- Supplementary Remarks.

Chapter 1: Sections 1.7 - 1.11

UNIT - III: VARIATIONAL PROBLEMS WITH MOVING BOUNDARIES 18 Hours

Functional of the form – Variational Problem with a Movable Boundary for a Functional Dependent on Two Functions- One-Sided Variations- Reflection and Refraction of Extremals- Diffraction of Light Rays.

Chapter 2: Sections 2.1 - 2.5

UNIT - IV: SUFFICIENT CONDITIONS FOR AN EXTREMUM

18 Hours

Field of Extremals- Jacobi Condition- Weierstrass Function- Legendre Condition- Second Variation- Canonical Equations and Variational Principles.

Chapter 3: Sections 3.1 - 3.6

UNIT - V: SUFFICIENT CONDITIONS FOR AN EXTREMUM (CONTINUED) 18 Hours

Complementary Variational Principles - Poisson Bracket - Contact Transformations- The Hamilton-Jacobi Equation- Clairaut's Theorem- Noether's Theorem.

Chapter 3: Sections 3.7 - 3.12

DISTRIBUTION OF MARKS: THEORY 80% AND PROBLEMS 20%

TEXT BOOK

| S.NO | AUTHORS | TITLE | PUBLISHERS | YEAR OF PUBLICATION |
|------|-----------|--|--------------------------------------|---------------------|
| 1. | A.S.Gupta | Calculus of Variations with Applications, 14 th Edition | PHI Learning Private Limited, Delhi. | 2019 |

REFERENCE BOOKS

| S.NO | AUTHORS | TITLE | PUBLISHERS | YEAR OF PUBLICATION |
|------|----------------------------|------------------------|---------------------------|---------------------|
| 1. | I.M. Gelfand and S.V.Fomin | Calculus of Variations | Prentice Hall, New Jersey | 1963 |
| 2. | Weinstock | Calculus of Variations | McGraw Hill | 2000 |

WEB RESOURCES

1. Calculus of Variations
2. Introduction to the Calculus of Variation

TEACHING METHODOLOGY

1. Class room teaching
2. Giving Assignments for all units
3. Discussions
4. Home test
5. PPT presentation

SYLLABUS DESIGNER

1. Mrs. S.Baby Shalini, Assistant Professor of Mathematics.
2. Mrs.R.Ramya, Assistant Professor of Mathematics.