## REAL ANALYSIS - I

| Semester | Subject Code | Category | Lecture | | Theory | | Practical | Credits |
|---|---|---|---|---|---|---|---|---|
| I | 21CPMA1B | Core | Hrs/ week | Hrs/ Sem | Hrs/ week | Hrs/ Sem | 0 | 5 |
| | | | 6 | 90 | 6 | 90 | | |

**COURSE OBJECTIVES:**

The students will be able to

1. This course aims to provide students with the specialist knowledge necessary for basic concepts in Real Analysis. More precisely, it strives to enable students to learn basic concepts about functions of bounded variation, grasp basic concepts about the total variation, learn about Riemann - Stieltjes integrals, sequences and series of functions.
2. We introduce a stronger notion of convergence of functions than point wise convergence, called uniform convergence. The difference between point wise convergence and uniform convergence is analogous to the difference between continuity and uniform continuity.

**COURSE OUTCOMES:**

On the successful completion of the course, the students will be able to

| CO Number | CO Statement | Knowledge Level (K1-K4) |
|---|---|---|
| **CO1** | Determine the algebraic properties as well as more abstract properties such as realizing that every function of bounded variation can be written as the difference of two increasing functions. | K4 |
| **CO2** | Recognize the concept of the terms of Riemann integrability and the Riemann-Stieltjes integrability of a bounded function and proved a selection of theorems concerning integration. | K2 |
| **CO3** | Determine the Mean value theorems for Riemann - Stieltjes Integrals depending on a parameter. | K2 |
| **CO4** | Both infinite series and infinite products could potentially be helpful in the area of approximation of functions. Infinite series represent a traditional instrument in contemporary mathematics. | K2 |
| **CO5** | Illustrates the effect of uniform convergence on the limit function with respect to continuity, differentiability, and integrability of functions defined on subsets of the real line. Illustrate the derivatives of higher order and differentiation of integral. | K3 |

*Knowledge Level: K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze.*

**MAPPING WITH PROGRAMME OUTCOMES:**

| COS | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 |
|---|---|---|---|---|---|---|
| **CO1** | S | S | M | S | S | M |
| **CO2** | M | S | S | S | S | S |
| **CO3** | S | M | S | M | S | S |
| **CO4** | M | S | S | S | S | S |
| **CO5** | S | S | S | S | S | S |

S- Strong; M-Medium; L-Low

**UNIT- I - FUNCTIONS OF BOUNDED VARIATION** **18 Hours**

Introduction - Properties of monotonic functions - Functions of bounded variation - Total Variation - Additive property of total variation - Total variation on [a,x] as a function of x - Functions of bounded variation expressed as the difference of increasing functions - Continuous functions of bounded variation-Curves and paths- Rectifiable paths and arc length - Additive and Continuity Properties of arc length-Equivalence of paths , Change of parameter.

**Section: 6.1 - 6.12**

**UNIT- II - THE RIEMANN- STIELTJES INTEGRAL** **18 Hours**

Introduction – Notation – The definition of the Riemann-Stieltjes integral – Linear properties – Integration by parts – Change of variable in a Riemann-Stieltjes integral – Reduction to a Riemann Integral – Euler's summation formula – Monotonically increasing integrators, Upper and lower integrals – Additive and linearity properties of upper and lower integrals – Riemann's condition – Comparison theorems.

**Section 7.1 - 7.7 and 7.10 - 7.14**

**UNIT - III - THE RIEMANN-STIELTJES INTEGRAL** **18 Hours**

Integrators of bounded variation – Sufficient conditions for existence of Riemann-Stieltjes integrals - Necessary conditions for existence of Riemann-Stieltjes integrals – Mean value Theorems for Riemann – Stieltjes integrals – The integral as a function of the interval – Second fundamental theorem of integral calculus – Change of variable in a Riemann integral – Second Mean-Value Theorem for Riemann integrals – Riemann-Stieltjes integrals depending on a parameter – Differentiation under the integral sign -Interchanging the order of integration – Lebesgue's criterion for existence of Riemann integrals.

**Section: 7.15 - 7.26**

**UNIT- IV - INFINITE SERIES AND INFINITE PRODUCTS** **18 Hours**

Absolute and conditional convergence- Dirichlet's test and Abel's test- Rearrangement of series- Reimann's Theorem on Conditionally convergent series -Double sequences – Double series – Rearrangement theorem for double series – A sufficient condition for equality of iterated series – Multiplication of series – Cesarosummability – Infinite products.

**Section: 8.8 - 8.26**

**UNIT- V- SEQUENCE OF FUNCTIONS** **18 Hours**

Point wise convergence of sequence of functions – Examples of sequences of real -valued functions – Definition of uniform convergence – Uniform convergence and continuity – The Cauchy condition for uniform convergence – Uniform Convergence of infinite series of functions - Uniform convergence and Reimann- Stieltjes integration – Nonuniformly convergent sequence that can be integrated term by term– Uniform convergence and differentiation – Sufficient conditions for uniform convergence of a series – Uniform convergence and double sequences - Mean convergence -Power Series – Multiplication of power series – The Taylor's series generated by a function- Bernstein's theorem- The binomial series – Abel's limit theorem – Tauber's theorem

**Section: 9.1 - 9.6, 9.8 - 9.15 and 9.19 - 9.23**

**Distribution of Marks: Theory 100%**

**Text Books**:

| S.No | AUTHORS | TITLE | PUBLISHERS | YEAR OF PUBLICATION |
|---|---|---|---|---|
| 1 | Tom M. Apostol | Mathematical Analysis | Narosa Publishing House, New Delhi. | 2002 |

**Reference Books:**

| S.No | AUTHORS | TITLE | PUBLISHERS | YEAR OF PUBLICATION |
|---|---|---|---|---|
| 1 | Burkill, J.C | The Lebesgue Integral | Cambridge University Press | 1951 |
| 2 | Munroe, M.E | Measure and Integration | Wesley, Mass | 1971 |
| 3 | Roydon, H.L | Real Analysis | Company, New York | 1988 |
| 4 | Rudin, W and SavitaArora | Principles of Mathematical Analysis | McGraw Hill Company, New York | 1979. |
| 5 | Malik, S.C | Mathematical Analysis | Wiley Eastern Limited, New York | 1991 |

**Web sources:**

1. https://www.scribd.com/doc/19250862/Chap-07-Real-Analysis-Functions-of-Bounded-Variation
2. https://math.stackexchange.com/questions/206848/derivation-of-riemann-stieltjes-integral
3. http://www.springer.com/978-0-8176-8279-8
4. http://www.math.iitb.ac.in/~srg/courses/ma403-2008/uniconv.pdf
5. http://math.louisville.edu/~lee/ira/IntroRealAnal-ch09.pdf

**TEACHING METHODOLOGY:**

1. Black Board Teaching
2. Smart Board Teaching
3. Giving Assignments in each Unit.
4. Class Room Discussion and Seminars.
5. PPT Presentations.

**SYLLABUS DESIGNERS**

1. Mrs. Y.Vishnupriya, Assistant Professor of Mathematics.
2. Dr.B.Vijayalakshmi, Assistant Professor of Mathematics.