

REAL ANALYSIS – II

Semester	Subject Code	Category	Lecture		Theory		Practical	Credits
II	21CPMA2B	Core	Hrs/week	Hrs/Sem	Hrs/week	Hrs/Sem	0	5
			6	90	6	90		

COURSE OBJECTIVES

The students will be able to

- Introduce the concept of sequences and series of functions, Lebesgue measure and Lebesgue integration and to have a working knowledge on Multi-variable calculus.
- Measure on the real line, Lebesgue measurability and integrability, Fourier Series and Integrals.

COURSE OUTCOMES:

On the successful completion of the course, the students will be able to

CO Number	CO Statement	Knowledge Level (K1-K4)
CO1	This course will develop an appreciation of the basic concepts of measure theory. Able to learn advanced the Lebesgue measure and Lebesgue integral with related problems.	K2
CO2	Demonstrate understanding of the statement and proofs to Study the Stone-Weierstrass theorem and its applications.	K3
CO3	Understanding of the basic concepts underlying the definition of the general Lebesgue integral and Apply the theory of the course to solve a variety of problems at an appropriate level of difficulty	K2
CO4	Describe the Riemann integral and convergence of measure.	K3
CO5	Apply the concept of Mean-value theorem for differentiable functions.	K4

Knowledge Level: K1 – Remember; K2 – Understand; K3 – Apply; K4 – Analyze.

MAPPING WITH PROGRAMME COURSE OUTCOMES:

COS	PO1	PO2	PO3	PO4	PO5	PO6
CO1	S	S	M	M	S	M
CO2	M	S	M	S	M	S
CO3	S	S	S	M	M	S
CO4	S	M	M	M	M	M
CO5	S	S	S	S	S	M

S- Strong; M-Medium; L-Low

UNIT- I - THE LEBESGUE INTEGRAL**18 Hours**

Introduction – The integral of a step function – Monotonic sequences of step functions – Upper functions and their integrals – Riemann integrable functions as examples of upper functions – The class of Lebesgue-integrable functions on a general interval – Basic properties of the Lebesgue integral – Lebesgue integration and sets of measure zero- The Levi monotone convergence theorems- The Lebesgue dominated convergence theorem – Lebesgue integrals on unbounded intervals as limits of integrals as on bounded intervals- Improper Riemann integrals.

Chapter 10: Section 10.1 to 10.13**UNIT- II - MEASURES THEORY AND INTEGRATION****18 Hours**

Measurable functions – Continuity of function defined by Lebesgue integrals- Differentiation under the integral sign – Interchanging the order of integration- Measurable sets on the real line – The Lebesgue integral over arbitrary subsets of \mathbb{R} – Lebesgue integrals of complex- valued functions – Inner products and norms – The set $L^2(I)$ of square- integrable functions – The sets $L^2(I)$ as a semimetric space – A convergence theorem for series of functions in $L^2(I)$ – The Riesz-Fischer theorem.

Chapter 10: Section 10.14 to 10.25**UNIT - III-FOURIER SERIES AND FOURIER INTEGRALS****18 Hours**

Introduction – Orthogonal system of functions – The theorem on best approximation – The Fourier series of function relative to an orthonormal system – Properties of Fourier Coefficients – The Riesz – Fischer Theorem – The convergence and representation problems for trigonometric series – The Riemann – Lebesgue Lemma – The Dirichlet Integrals – An Integral representation for the partial sums of a Fourier series – Riemann's localization theorem- Sufficient conditions for convergence of a Fourier Series at a particular point – Cesaro summability of Fourier series – Consequences of Fejer's theorem – The Weierstrass approximation theorem.

Chapter 11: Section 11.1 to 11.15**UNIT- IV - MULTIVARIABLE DIFFERENTIAL CALCULUS****18 Hours**

Introduction – The directional derivative – Directional derivatives and continuity – The total derivative – The total derivative expressed in terms of partial derivatives – An application to complex- valued functions- The matrix of linear function – The Jacobian matrix – The chain rule

– Matrix form of the chain rule – The Mean-Value Theorem for differentiable functions – A sufficient condition for differentiability – A sufficient condition for equality of mixed partial derivatives – Taylor’s formula for functions from \mathbb{R}^n to \mathbb{R}^1

Chapter 12: Section 12.1 to 12.14

UNIT- V - IMPLICIT FUNCTIONS AND EXTREMUM PROBLEMS

18 hrs

Introduction-Functions with nonzero Jacobian determinant – The inverse function theorem – The implicit function theorem – Extrema of real valued functions of one variable- Extrema of real valued functions of several variables – Extremum problems with side conditions.

Chapter 13: Section 13.1 to 13.7

DISTRIBUTION OF MARKS: THEORY 100%

TEXT BOOK:

S.NO	AUTHORS	TITLE	PUBLISHERS	YEAR OF PUBLICATION
1.	Tom M. Apostol	Mathematical Analysis	Narosa Publishing House, New Delhi.	2002

REFERENCE BOOKS:

S.NO	AUTHORS	TITLE	PUBLISHERS	YEAR OF PUBLICATION
1	Burkill, J.C.	The Lebesgue Integral	Cambridge University Press	1951.
2	Malik, S.C. and Savita Arora	Mathematical Analysis	Wiley Eastern Limited	1991.
3	Rudin, W	Principles of Mathematical Analysis	McGraw Hill Company	1979.

WEB SOURCES:

1. <https://www.iiserkol.ac.in/Measure-Integration-notes.pdf>
2. [https://www.amazon.in/Lebesgue Integration-notes.pdf](https://www.amazon.in/Lebesgue-Integration-notes.pdf)

TEACHING METHODOLOGY

1. Class room Teaching
2. Assignments
3. Seminars
4. Discussions
- 5 .PPT Presentations

SYLLABUS DESIGNER:

1. Ms.Y.Vishnupriya, Assistant Professor of Mathematics
2. C.Revathi, Assistant Professor of Mathematics