Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

JUNE – 2022 **19CMA6A**

REAL ANALYSIS - II

Time: 3 Hours **Max. Marks: 75**

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. *Is* $[0,1] \cup [2,3]$ *connected? Justify.*
2. *Define totally bounded.*
3. *Give an example of a set which is complete but not totally bounded.*
4. *If* $u = \{u_n\}_{n=1}^{\infty} \in l^2, let\ Tu = \{u_n/2\}_{n=1}^{\infty}$ *then T is contraction on M find the value of* α.
5. *Define a statement almost every point of* $[a,b]$.
6. *Compare subdivision and refinement of* $[a,b]$.
7. *Verify Rolle's theorem for* $g(x) = 1 - |x| \quad (-1 \le x \le 1)$
8. *Give an example of the equation which has at least one root between 0 and 1.*
9. *Define Taylor's series.*
10. *Expand* $(1+x)^m$ *where m is not a nonnegative integer and* $|x| < 1$.

SECTION – B (5 x 5 = 25)

Answer ALL the questions.

11. *(a) If* $\mathcal{F}$ *is any family of closed set of Metric space M, then prove that* $\bigcap_{f \in \mathcal{F}} F$ *is also closed set.*

(Or)

(b) If E is any subset of Metric Space M then prove E is closed.

12. *(a) State and prove generalization of nested interval theorem.*

(Or)

(b) If M is compact then prove that M has Heine - Borel Property.

13. *(a) If each of the subsets* $E_1 E_2 \ldots$ *of* R^1 *is of measure zero then prove that* $\bigcup_{n=1}^{\infty} E_n$ *is also of measure zero.*

(Or)

(b) If $f \in \mathfrak{R}[a,b]$ *and* λ *is any real number then* $\lambda f \in \mathfrak{R}[a,b]$ *then prove that* $\int_a^b \lambda f = \lambda \int_b^a f$.

14. *(a) State and prove Rolle's theorem.*

(Or)

(b) State and prove second fundamental theorem of calculus.

15. *(a) Establish and derive the Taylor's formula with the Lagrange form of remainder.*

(Or)

(b) Establish and derive the Taylor's formula with the Cauchy form of remainder.

SECTION – C (3 x 10 = 30)

Answer any THREE of the following questions.

16. *Let $< M, \rho >$ be a metric space. Show that the subset A of M is totally bounded if and only if every Cauchy sequence of point of A contains a Cauchy subsequence.*
17. *State and prove Picard fixed point theorem.*
18. *Let f be bounded function on [a,b] then show that every upper sum for f is greater than or equal to every lower sum of f.*
19. *State and prove first fundamental theorem of calculus.*
20. *Derive Taylor's formula with integral form of remainder.*

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