Reg.No. | | | | | | | | | | | |

D.K.M.COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

JUNE – 2022 **19CPMA4C**

FUNCTIONAL ANALYSIS

Time : 3 Hrs **Max. Marks : 75**

SECTION-A (5x6=30)

Answer ALL the questions.

1. *(a) State and prove Holder's inequality.*

   (Or)

   (b) Prove that if T is a continuous linear transformation of a normed linear space N into normed linear space N' and if M is its null space such that T induces a natural linear transformation T'on N/M into N' and $\|T'\|=\|T\|$.

2. *(a) Prove that (i)* $\{0\}^{\perp}=H$ *and* $H^{\perp}=\{0\}$ *(ii)* $S\cap S^{\perp}\subseteq \{0\}$ *(iii)* $S_1\subseteq S_2 \Rightarrow S_1^{\perp} \supseteq S_2^{\perp}$.

   (Or)

   (b) Prove that a non-empty subset X of a normed linear space is bounded iff f(x) is bounded set of numbers for each f is N^{*}.

3. *(a) Prove that if T is an operator on a Hilbert space H then (Tx,x)=0,* $\forall x$ *in H then T=0.*

   (Or)

   (b) State and prove Riez representation theorem.

4. *(a) Prove that every element of x for which* $\|x-1\|<1$ *is regular and the inverse of such element is given by the formula* $x^{-1}=1+\sum_{n=1}^{\infty}(1-x)^n$

   (Or)

   (b) Prove that if zero is the only topological divisor of zero in A then A=₵.

5. *(a) Prove that if* F_1 *and* F_2 *are multiplicative functional on A with the same null space M then* $F_1= F_2$

   *(Or)*Type equation here.

   (b) Prove that the maximal ideal space M is a compact Hausdroff space.

SECTION-B (3x15=45)

Answer any THREE of the following questions.

6. *State and prove Hahn Banach theorem.*
7. *State and prove Uniform boundedness theorem.*
8. *Prove that an operator T on H is unitary iff it is an isometric isomorphism of H onto itself.*
9. *Prove that the boundary of S is a subset of Z.*
10. *State and prove Gelfand mapping theorem.*

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