Reg.No : | | | | | | | | | | | |

D.K.M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1

SEMESTER EXAMINATIONS

JUNE – 2022 21CMA2A

CALCULUS

Time: 3 Hours **Max. Marks: 75**

SECTION – A (10 x 2 = 20)

Answer ALL the questions.

1. *If y = acos5x + bsin5x show that* $\frac{d^2y}{dx^2} + 25y = 0$.
2. *Find the n^th derivative of sin(ax + b).*
3. *Prove that the radius of curvature at the point (0, 1) on the curve* $y = e^x$ *is* $2\sqrt{2}$.
4. *Find the p – r equation of* $r = a\theta$.
5. *Mention the two properties of evolutes.*
6. *Find the asymptotes of the curve (x – y)(x + y)(x + 3y – 7) – (2x – 3y + 1) = 0 .*
7. *Evaluate* $\int_0^{\frac{\pi}{2}} cos^7 x dx$.
8. *If* $x = u^2 - v^2, y = 2uv$ *find* $\frac{\partial(x,y)}{\partial(u,v)}$.
9. *Evaluate* $\int_0^{\frac{\pi}{2}} \int_0^a dr\, d\theta$.
10. *Evaluate* $\int_0^2 \int_1^3 \int_1^2 xzdzdydx$.

SECTION – B (5 x 5 = 25)

Answer ALL the questions.

11. *(a) If* $y = (tan^{-1}x)^2$ *show that* $(1 + x^2)^2 y_2 + 2x(1 + x^2)y_1 = 0$.

(Or)

(b) Find the n^th derivative of $e^{3x} sinx\, sin2x\, sin3x$.

12. *(a) Prove that the radius of curvature at the point* $(acos^3\theta, asin^3\theta)$ *on the curve* $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ *is* $3a\, sin\theta\, cos\theta$.

(Or)

(b) Show that the radius of curvature at any point on the equiangular spiral $r = ae^{\theta cot\alpha}$ *is* $r = cosec\alpha$.

13. *(a) Find the equation of the evolute of the curve* $x = a(cos\theta + \theta sin\theta), y = a(sin\theta - \theta cos\theta)$.

(Or)

(b) Find all the asymptotes of $(x - y)^2(x^2 + y^2) - 10(x - y)x^2 + 12y^2 + 2x + y = 0$.

14. *(a) Derive the reduction formula for* $\int_0^{\frac{\pi}{2}} sin^n x dx$.

(Or)

(b) Express $\int_0^1 x^m(1 - x^n)^p\, dx$ *in terms of gamma function and evaluate* $\int_0^1 x^5(1 - x^3)^{10}\, dx$.

15. *(a) Evaluate* $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{\sqrt{1+x^2+y^2}}$.

(Or)

(b) Evaluate $\iint xydxdy$ *over the region in the positive quadrant for which x + y = 1 .*

SECTION – C (3 x 10 = 30)

Answer any THREE of the following questions.

16. *Find the minimum of* $a^3x^2 + b^3y^2 + c^3z^2$ *with the condition* $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$.

17. *Prove that the radius of curvature at any point of the cycloid* $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$ *is* $4acos\frac{\theta}{2}$.

18. *Find the asymptotes of the curve* $y^3 - 2y^2x - yx^2 + 2x^3 + x^2 - 6xy + 5y^2 - 2y + 2x + 1 = 0$.

19. *Show that* $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}}$.

20. *Find the volume of the region bounded by the surfaces* $y^2 = 4ax, x^2 = 4ay$ *and the plane* $z = 0$ *and* $z = 3$.

* * * * * *