

D. K. M. COLLEGE FOR WOMEN (AUTONOMOUS), VELLORE-1
SEMESTER EXAMINATIONS
JUNE - 2022
DIFFERENTIAL GEOMETRY

21CPMA2D

Time: 3 Hours

Max. Marks: 75

SECTION – A (5 x 6 = 30)

Answer ALL the questions.

1. (a) State and prove Serret Frenet formula.

(Or)

(b) Find curvature and torsion of $\vec{r} = (u, u^2, u^3)$.

2. (a) Find the coefficient of the direction which makes an angle $\pi/2$ with direction whose coefficient of (m,n) .

(Or)

(b) Obtain the fundamental form of the sphere $\vec{r} = (a \sin u \cos v, a \sin u \sin v, a \cos u)$ and show that the parametric curves are orthogonal.

3. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = C$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = C$ for all values of E .

(Or)

(b) Derive an expression for kg .

4. (a) State and prove Rodrigues formula.

(Or)

(b) If there is a surface of minimum area passing through a closed space curve then prove that it is necessarily a minimal surface i.e. a surface of zero mean curvature.

5. (a) Prove that the only compact surfaces whose Gaussian curvature constant mean curvature constant are spheres.

(Or)

(b) Prove that in a closed region R of a surface of constant positive Gaussian curvature without umbilics, the principal curvatures take their extreme values at the Boundary.

SECTION – B (3 x 15 = 45)

Answer any THREE of the following questions.

6. State and prove Fundamental existence theorem for space curves.

7. A helicoid is generated by the screw motion of the straight line which meets the axis at an angle α . Find the orthogonal Trajectories of the generators and also find the metric of the surface.

8. State and Prove Gauss – Bonnet theorem.

9. Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

10. State and prove Fundamental Existence theorem for surfaces.